Cyclic Motion Planning of Redundant Robot Arms: Simple Extension of Performance Index May Not Work

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Abstract

In this paper, multiple types of performance indices (termed, an original one and its simple extensions) are investigated for online cyclic motion planning of redundant manipulators, which aims at remedying a so-called joint-angle-drift problem. In addition, physical constraints such as joint limits and joint velocity limits are taken into consideration in these related scheme-formulations, and these schemes could finally be rewritten as a quadratic program (QP). These cyclic-motion-planning (CMP) schemes are then applied and simulated based on three different types of robot arms, which shows that the original one is effective, whereas its simple extensions may not work applicably.

1. Introduction

In recent years, robotic researchers have focused on solving a variety of tasks requiring sophisticated motion in complex environments [¹][²], for instance, working in hazardous or rough-and-tumble situations, carrying radioactive materials or heavy objects, doing cyclic work, and exploring unpredictable regions. As we may know, a robot arm with just enough degrees of freedom (DOF) (for a specific task) may not have the ability to achieve alternative goals while tracking the specified end-effector trajectory. Moreover, the end-effector motion may not be performed accurately or even cannot be fulfilled, if a robot does not have sufficient DOF. These problems have motivated robot-practitioners to improve the functionality and flexibility of robot arms by considering the redundancy mechanism. However, a fundamental issue in operating the redundant robot systems is the inverse-kinematic problem [³]. For a redundant robot arm performing a specified end-effector task, multiple solutions (or even an infinite number of solutions) exist. In this sense, the redundancy of joint motion may complicate the robots’ planning-and-control problem considerably, in addition to the kinematic and dynamic nonlinearities. To make good use of the redundancy, various computational schemes have thus been developed. The conventional solution of inverse-kinematics is pseudoinverse-based [⁴]. Our nearly ten-year research [²][⁴] shows that inverse-kinematic problems might be solved more favorably by QP techniques.

In this paper, we pay attention to the cyclic motion planning of different robot manipulators by testing more performance indices. In other words, the so-called joint angle drift problem that we are focusing on in this paper, could be defined as follows: “When the end-effector traces a closed path in its workspace, the joint variables may not return back to their initial values after completing such an end-effector task”. Theoretical analysis and simulation results, however, show that “the original performance-index is effective, whereas its simple extensions may not work applicably” — a philosophical conclusion (i.e., people should not take things and/or their minor extensions for granted).

2. Performance-indices and formulations

The relation \( f(\cdot) \) between the end-effector position-and-orientation vector \( r(t) \in \mathbb{R}^m \) and joint variable vector \( \theta(t) \in \mathbb{R}^n \) for redundant manipulators could be written as

\[
    r = f(\theta). \quad (1)
\]

By differentiating (1), we have the point-wise linear relation between the Cartesian velocity \( \dot{r} \) and joint velocity \( \dot{\theta} \):

\[
    J(\theta)\dot{\theta} = \dot{r}, \quad (2)
\]

where \( J(\theta) := \partial f(\theta)/\partial \theta \) is the Jacobian matrix. In redundant manipulators, equations (1) and (2) are under-determined and admit an infinite number of solutions.

The conventional solution to such an inverse-kinematic problem (i.e., given \( r \), to solve \( \theta \)) is the pseudoinverse-type solution. However, it is shown in [⁵] and many other references that the pseudoinverse-type solution may not be...
cyclic (or termed, repetitive). To make the kinematic solution cyclic, the minimization of the joint displacement between current and initial states can be exploited \cite{1,2,6,7}. In the formulation, the performance index (termed the original performance index in this paper) is

\[ (\dot{\theta} + c)^T (\dot{\theta} + c)/2 \]

with \( c = \lambda (\theta - \theta(0)) \),

where \( \lambda > 0 \) is used to scale the magnitude of the manipulator response to such joint displacements, and \( \theta(0) \) is the initial state. Moreover, in our work \cite{1,2,7}, in view of the fact that almost all robot arms are constrained physically by their joint limits \([\theta^-, \theta^+]\) and joint velocity limits \([\dot{\theta}^-, \dot{\theta}^+]\), we have the following more realistic CMP formulation:

\[ \begin{align*}
\text{minimize} & \quad (\dot{\theta} + c)^T (\dot{\theta} + c)/2, \\
\text{subject to} & \quad J(\dot{\theta})\dot{\theta} = \dot{r}, \\
& \quad c = \lambda (\theta - \theta(0)), \\
& \quad \theta^- \leq \theta \leq \theta^+, \\
& \quad \dot{\theta}^- \leq \dot{\theta} \leq \dot{\theta}^+. \end{align*} \]

where coefficients \( W(1) := I \) and \( c = \lambda (\theta - \theta(0)) \), and the \( i \)th elements of new bounds \( \xi^- \) and \( \xi^+ \) can be defined respectively as \( \max\{\theta^-_i, \mu(\theta^-_i - \theta_i)\} \) and \( \min\{\theta^+_i, \mu(\theta^+_i - \theta_i)\} \), with intensity coefficient \( \mu > 0 \) used to scale the feasible region of \( \dot{\theta} \). For presentation convenience, \( (8)-(10) \) is called in this paper the original CMP scheme.

Facing the success of CMP scheme \( (8)-(10) \) \cite{1,2,7}, we may think of a new performance index for such drift-free purposes; i.e., the performance index and scheme below:

\[ \begin{align*}
\text{minimize} & \quad \dot{\theta}^T W(2) \dot{\theta}/2, \\
\text{subject to} & \quad J(\dot{\theta})\dot{\theta} = \dot{r}, \\
& \quad c = \lambda (\theta - \theta(0)), \\
& \quad \theta^- \leq \theta \leq \theta^+, \\
& \quad \dot{\theta}^- \leq \dot{\theta} \leq \dot{\theta}^+. \end{align*} \]
by forcing\( e^{-i\theta} W f \)actor tracking a circular path\( 3^2 \lambda T(0) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} g \), which, however, do not
in radians. \( 0 \), or
\( 0 \). As
\( 0 \) or preferably by using recurrent neural networks (e.g.,
the L VI-based primal-dual neural networks
using optimization routines (e.g., MATLAB “Quadprog”
or by computer-simulations for the effectiveness.
Before ending this section, it is worth pointing out here
that QP (8)-(10) and (11)-(13) could be solved readily by
using optimization routines (e.g., MATLAB “Quadprog”
preferably by using recurrent neural networks (e.g.,
the LVI-based primal-dual neural networks\(^1\)[4]).

3. Computer-simulation verification

In this section, both original and new CMP schemes are
simulated and tested based on three different types of robot
arms to verify the schemes’ effectiveness.

3.1. PA10 robot arm

In this subsection, computer simulations are performed
based on a 7-DOF PA10 robot arm, of which the parameters
and limits are mentioned in\([1][2][7]\). Two simulation ex-
amples are shown as follows; i.e., a circular path-following
example and a straight-line path-following example.

3.1.1 Circular-path following

In this circular example, the motion trajectory of the PA10
end-effector is expected to be a circle with radius 0.2m
and the revolute angle about \( x \)-axis being \( \pi/6 \)rad. The
task duration time is 10 seconds, and the initial joint state
\( \theta(0) = [0, \pi/4, 0, 2\pi/3, 0, -\pi/4, 0]^T \) in radians.

Firstly, let us consider the situation of no drift-free cri-
terion; i.e., the performance index is \( \hat{\theta}^T \hat{\theta}/2 \) by forcing \( \lambda = 0 \)
in (8). As seen from Figure 1(a) with \( \mu = 1 \), the solution
is not repetitive in the sense that the final state of the PA10
arm does not coincide with its initial state. Therefore, such
a solution is thought to have a joint-angle-drift problem.

Secondly, let us consider the inclusion of the original
CMP performance index (8) with \( \lambda = 4 \) and \( \mu = 1 \). Figure
1(b) shows the simulated result, which demonstrates that
the solution is repetitive [as the final and initial PA10 states
coincide well with each other].

Thirdly, let us consider the new performance index (11)
and its corresponding scheme (11)-(13) with \( \mu = 1 \). As
seen from Figure 1(c), there evidently still exists a joint-
angle-drift problem. In other words, in this PA10 circular
example, new performance index (11) can not achieve the
CMP purpose, though it appear to be capable.
3.1.2 Straight-line path following

In this second example of PA10, the motion trajectory of the end-effector is to move forwards and then backwards along a straight-line segment. The task duration is 7.0 seconds, the line-segment length is 2.5m, and the initial state $\theta(0) = [\pi/4, \pi/4, \pi/4, \pi/4, \pi/4, 0]^T$ rad. Angles of the desired straight-line segment making with XY, YZ and ZX planes are $\pi/4$ rad, $\pi/6$ rad and $\pi/6$ rad, respectively.

Firstly, let us consider the situation of no drift-free criterion and $\mu = 4$. The result is shown in Figure 2(a), and evidently the PA10’s final state does not coincide with its initial. That is, a so-called joint-angle-drift problem occurs.

Secondly, let us consider the inclusion of the original CMP performance index (8) with $\lambda = 4$ and $\mu = 4$. Figure 2(b) shows the result; i.e., the solution is now repetitive in the sense that the final PA10 state equals the initial state.

Thirdly, let us consider the new performance index (11) and its corresponding scheme (11)-(13) with $\mu = 4$ in this PA10 straight-line example. The result is shown in Figure 2(c). Evidently, such a new performance index could not be used to achieve the CMP purpose, and its performance is similar to that of $\dot{\theta}^T \hat{\theta}/2$.

3.2 PUMA560 robot arm

In this subsection, computer-simulations are conducted based on PUMA560 robot arm [2][4][8] to verify the above presented theoretical analysis as well as the PA10 observations [e.g., the new performance index (11) may not work practically]. In addition, it is worth noting that the QP solution in all simulations of this paper is actually solved via the LVI-based primal-dual neural network [1][4].

3.2.1 Circular-path following

In this example, the end-effector of the PUMA560 arm is to move along a circle in the 3-dimensional workspace. Figure 3(a) shows the result by using usual performance index $\dot{\theta}^T \hat{\theta}/2$, Figure 3(b) shows the result by using original CMP performance index (8). Evidently, by exploiting the original CMP performance index (8), the cyclic motion planning cloud be achieved (for the PUMA560 circular example).

For comparison and illustration, the new performance index (11) is also exploited with joint physical limits considered. However, as Figure 3(c) shows, such a simulated solution is not cyclic (or to say, not repetitive).
3.2.2 Straight-line path following

In this example, the end-effector of the PUMA560 arm is expected to move forwards and then backwards along a straight-line segment. Firstly, let us consider the situation of no drift-free criterion. As we can see from Figure 5(a), the solution (actually, as we observed, many of the inverse-kinematic solutions) has a joint-angle-drift phenomenon.

Secondly, let us consider the inclusion of the original CMP performance index (8). As we see from Figure 5(b), the solution is now cyclic, in view of the fact that the final PUMA560 state coincides with its initial state.

Thirdly, for comparison and illustration, we can take the new performance index (11) into simulation, and the simulated result is shown in Figure 5(c). Consequently, we can see again that the solution synthesized by (11) is not cyclic, and, in addition, it is similar to that synthesized by $\dot{\theta}^T \dot{\theta}/2$.

3.3 Four-link planar robot arm

Different from the above two examples of 3-dimensional robot manipulators (where PA10 is kinematically redundant, whereas PUMA560 could be functionally redundant), in this example we take a planar robot arm (i.e., a four-link planar robot arm) [2][6] for another example. Figure 5(a) illustrates the simulation result synthesized by usual performance index $\dot{\theta}^T \dot{\theta}/2$ (and with joint physical limits considered as well), Figure 5(b) illustrates the simulation result synthesized by original CMP performance index (8), and Figure 5(c) illustrates the simulation result synthesized by new performance index (11). In addition, their corresponding joint profiles are shown in Figure 6.

These simulation results demonstrate again that the original CMP scheme (8)-(10) is effective on cyclic robot-motion planning, whereas its simple extensions [e.g., the new scheme (11)-(13)] may not work applicably. Our further trial-and-error simulation-results show that using new performance index $c^T \dot{\theta}$ could achieve the CMP objective of initial and final states being equal (but “slowly” and with a chattering phenomenon). Moreover, using new performance index $\dot{\theta}^T W^{(2)} \dot{\theta}/2 + c^T \dot{\theta}$ could also work but unfortunately with no theoretical proof presented up to now.

4. Concluding remarks

This paper has investigated cyclic-motion-planning performance indices and schemes, with the verification based on three different types of robot arms. Consistent with our previous work [1][2][7], the original CMP scheme (8)-(10) works relatively perfectly, whereas the extended new performance-indices and schemes may not work as effectively as the original one does. In addition to [9], we might be happy at the philosophical conclusion that is confirmed here and might be used in arguments: “minor extension usually may not work, whereas, to make it work or to test whether it works, people may have to make a major effort”.

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